

The best way to get a handle on this problem, in my opinion, is to try a bunch of examples.
You might have noticed that *parity*, or evenness and oddness, has a role to play.

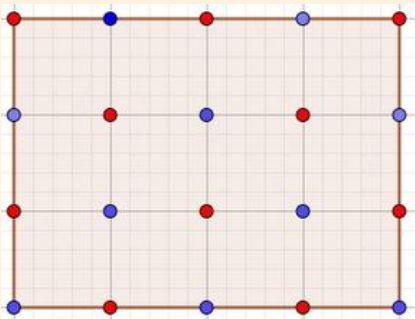
DIMENSIONS OF THE TABLE	CORNER THE BALL GOES IN
Odd by Odd	Top Right
Odd by Even	Bottom Right
Even by Odd	Top Left
Even by Even	Varies...

The “even by even” case seems stranger, but it can be dealt with by a simple observation: a 2 by 6 table is identical to a 1 by 3 table (seen from a closer vantage point, if you like). In general, whenever you have a table with two even numbers as its dimensions, you can divide them both by 2 until at least one of them is no longer even, and you haven’t changed the nature of the table.

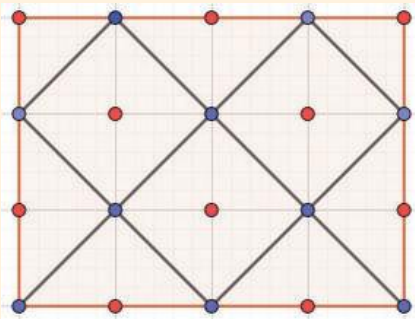
We can predict, according to this pattern, where the ball will wind up in the big tables:
26 by 47 - Top Left
35 by 99 - Top Right
501 by 998 - Bottom Right
600 by 10,000 - Bottom Right, because 600 by 10,000 is really just a scale model of a 3 by 50 table.

Now the real question: why does parity explain the path of the billiard balls?
I want to offer two explanations, one elegant, one illuminating.

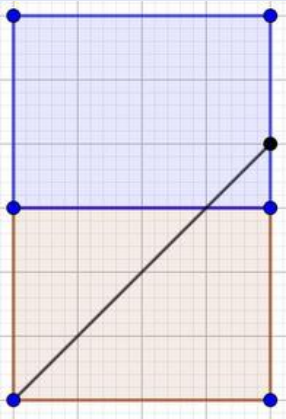
First, the elegant explanation. For any table, draw it on a grid – or lattice, as we sometimes say – and colour the grid points in a checker-board pattern.



Notice anything? The path the billiard ball takes can’t change from blue points to red points. That means we only need to figure out which of the four corners will be blue, or whatever colour matches the bottom left corner. The colour of the corners, as you can check, depends only on the parity of the dimensions of the table.

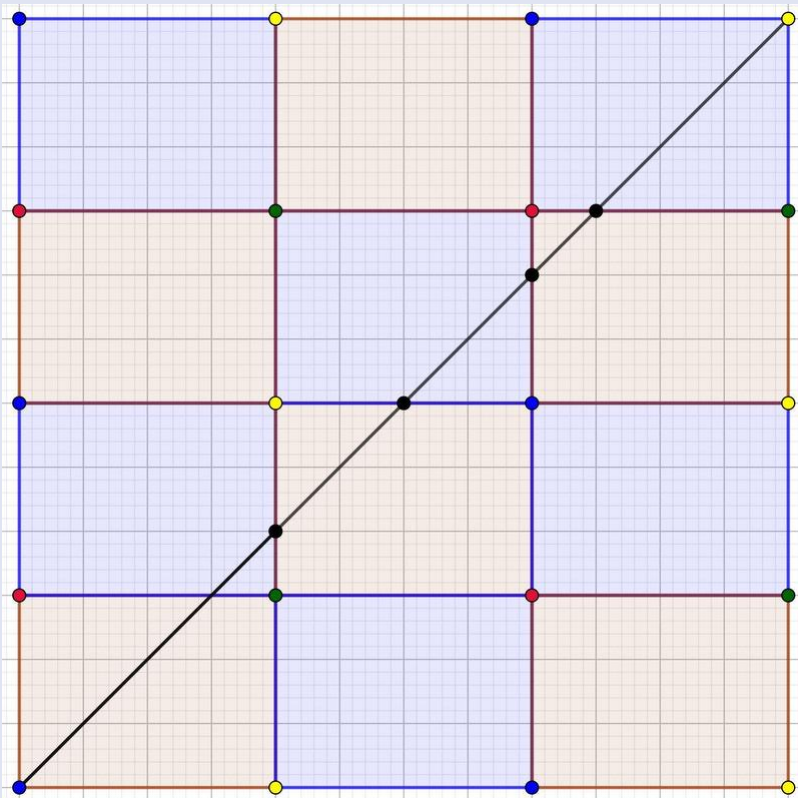


So that’s the elegant idea, and it’s easy for it to go by quickly. Here’s another take. Imagine the ball rolling toward the end of the table, and instead of bouncing off, imagine it passing “through the looking glass,” into a mirror image of the existing table.



Then it bounces again, into a mirror image. Keep expanding the mirror images, and we can image our ball on a straight line path. Not only that, its journey through these mirror image tables is precisely a mirror image of its real path. In particular, if we keep track of the corner, we can still predict correctly where it will end up.

In this case, using a 3 by 4 table, we can put 4 of them end to end vertically and 3 end to end horizontally. Colouring the corners of the original table to keep track, I can see that the mirror image straight-line path of the ball ends up in the bottom right corner.



What matters here? Only the parity of how many tables I had to stack to the right and how many I had to stack up. Evenness and oddness, once again, explains everything.

There are some details to work out in both these arguments, and I encourage doing so. Parity is strangely common in mathematics and in mathematical puzzles, and it is worth getting to know, for its simplicity and its power.