

A MATHEMATICIAN AT PLAY

Mysteries of continuous movement

Many things in our world move continuously. Continuous movement is one of those ideas that we all grasp naturally, but is actually quite subtle to define properly. **Daniel Finkel** introduces you to the intermediate value theorem before puzzling you with three interesting questions.

The key idea is that continuous motion doesn't leave gaps: if I run a race from start to finish, I must have been at all the points along the track at some point. There's a lot of mystery and paradox hiding in there. You pass through infinitely many points in a finite time? (Yep!) What was the last point you passed through before you hit the finish line? (There isn't one!)

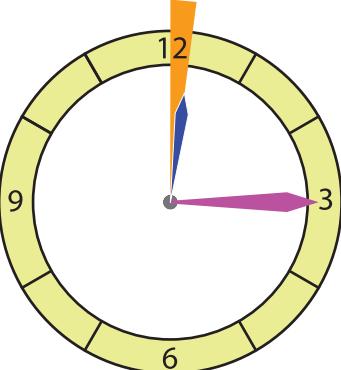
Lots of things aren't continuous too: scores in games or sports, for example, where the score jumps from 1 to 2 without ever being 1.5. But the movement of bodies or objects seems to be continuous, or close enough, that it's practically the same thing. If the temperature was 10 yesterday and 14 today, it must have been 12 and 13 and 13.5 somewhere between.

Even though this property of continuity – it is known as the intermediate value theorem – is simple to grasp, it can be used in some surprisingly powerful ways.

For example, it may come in handy thinking about today's puzzles:

PUZZLE 1

Do the minute and hour hands of an analog clock ever make a right angle?



(Note that at 12:15, the hands are not at a right angle. The area shaded orange shows how much it is lesser than 90 degrees.)

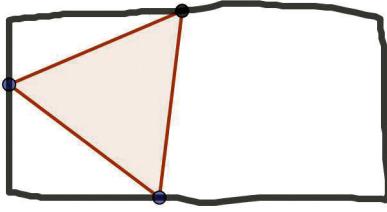
If no, how do you know? If yes, how many times per day?

PUZZLE 2

Draw an strange, squiggly, loop on paper – the only rule is that it has to start and stop at the same point, and be one continuous path with no gaps.



How can you prove that no matter what, there will always be three points on the loop that form an equilateral triangle?



PUZZLE 3

Prove that two points on opposite sides of the equator have the same temperature right now.

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how math is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling *Prime Climb* and *Tiny Polka Dot*.