A MATHEMATICIAN AT PLAY

Can you convince yourself about these COIN ROTATIONS?

We've all played with coins at one point of time or another. But have you ever tried rotating one around another, irrespective of their size? **Daniel Finkel** looks into coin rotations, starting out with three puzzles that seem simple, but are counter-intuitive...

In the United States, where I live, coins don't make much sense. Our nickels are worth less than dimes, but are larger and thicker. There's more value in the raw material of some pennies than the cent they're worth. It is all a little off-kilter.

Assuming you wanted the value of coins to be commensurate with their volume, a coin with twice the radius should be worth four times the value. That's because every time you double a dimension, you double the overall size, and doubling the radius effectively doubles both the length and height. If the coin was also twice as wide, it would use eight times as much metal.

There are lots of fun puzzles that would come up with a set of coins that were in this kind of 2 to 1 proportion. So let's assume we had coins like these. I'll call the small ones "alphas" (A) and the big ones "betas" (B).

The puzzles I want to share today are some of my favourites, because they are so counter-intuitive.
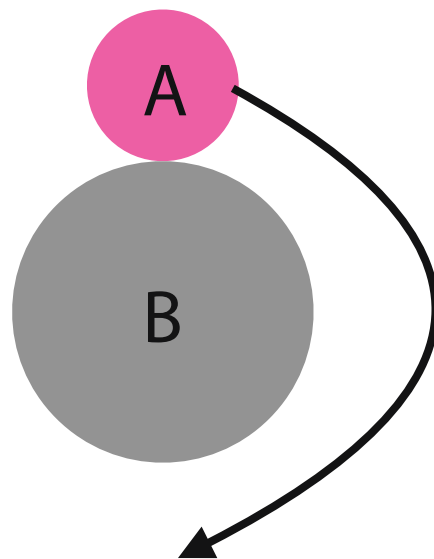
PUZZLE 1

Take two alpha coins, set one atop the other, and roll the top coin around the bottom one until it arrives back where it started. How many revolutions will it make on this journey?
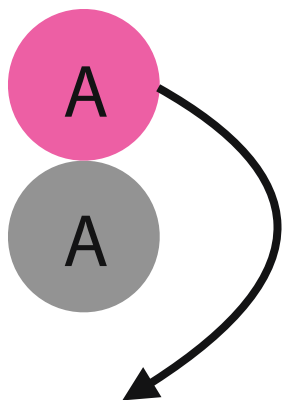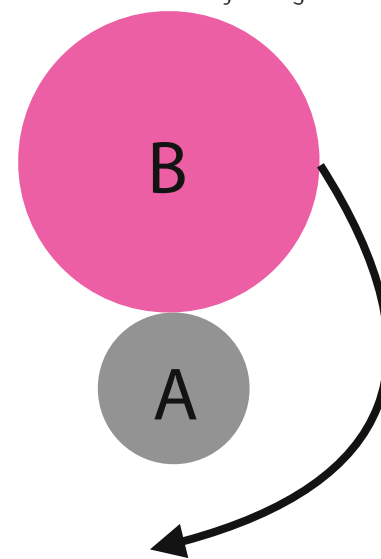
PUZZLE 2

Now put the alpha coin on top of the beta coin, and roll it until it arrives back where it started. How many revolutions will it make?

PUZZLE 3

Now consider what would happen if we asked the same questions about the beta coin rolling around the alpha coin. How many revolutions will the beta coin make on its journey?

The possibilities for extensions of this question are essentially limitless. Here's one that jumps to mind:

Research question: you have two coins with radii in an n:m ratio. Can you determine a formula for the number of revolutions one will turn as it rolls along the entire perimeter of the other?

You can write to Dan Finkel (dan@mathforlove.com) with your responses to the Research Question [subject: 16Play].

Dan Finkel is the founder of Math for Love, an organisation devoted to transforming how maths is taught and learned. He is the creator of mathematical puzzles, curriculum, and games, including the best-selling Prime Climb and Tiny Polka Dot.