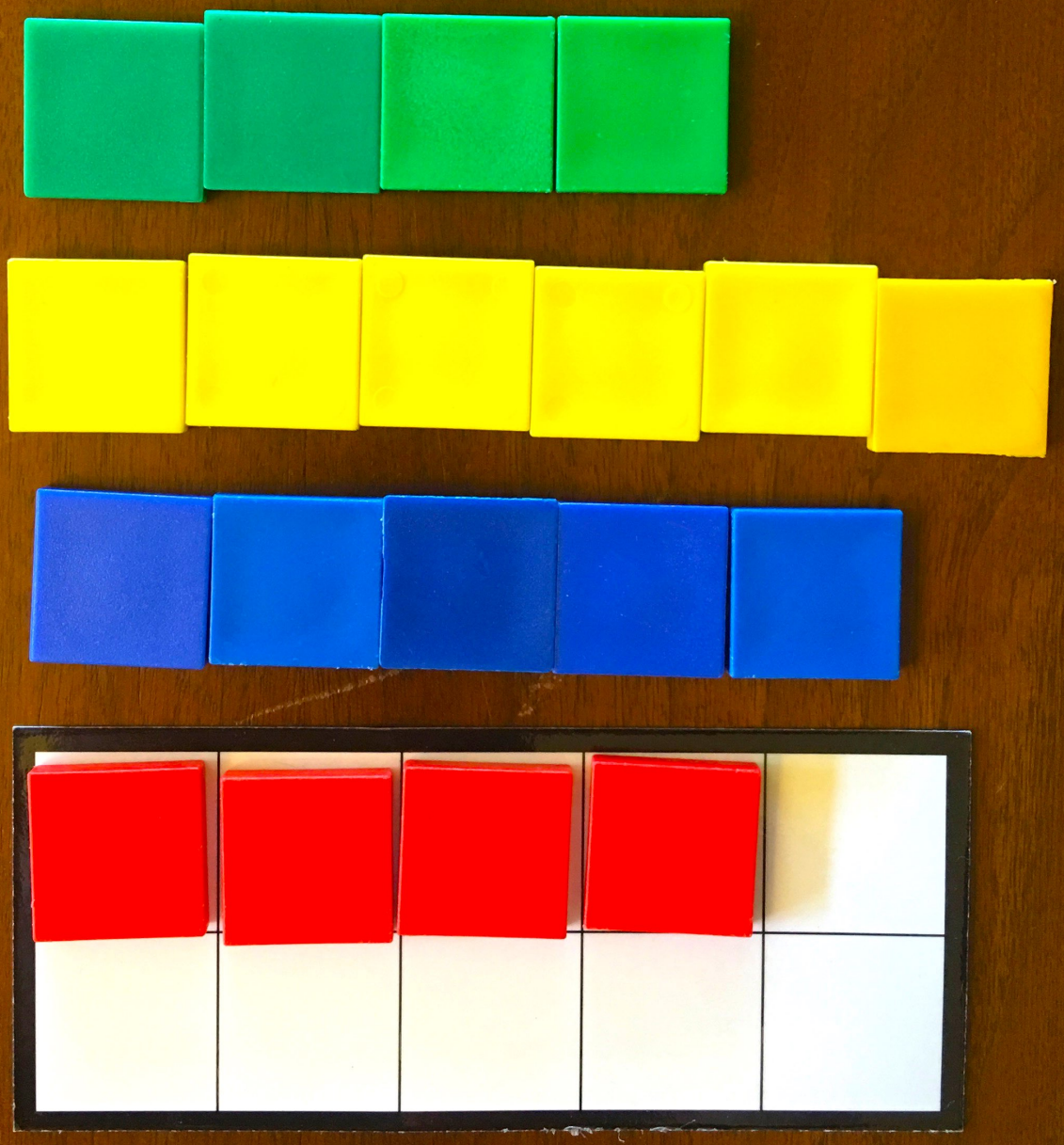
Sample

Math for Love

Kindergarten



by Dan Finkel & Katherine Cook

A word about using this book

This book was designed to support a summer math program lasting sixteen 75 - 90-minute days. With minimal adjustment it can be used for longer programs, programs with shorter classes, or other variations.

You can also use these activities to supplement a normal math class. There are enough activities to do something from this book 1-2 times a week for an entire school year. Most of the games can be played many times. Openers can be used in the first ten minutes of class. Games can be played for 5 - 30 minutes. Deeper tasks might be good for sparking your students' curiosity and digging in on a multi-day project. Use these in the way that works for you and your students.

The introduction in the following pages is worth reading, and can help get you started. We also have a video PD series to support this curriculum that should be helpful: mathforlove.com/video/math-for-love-video-pd.

Enjoy!

A word about the copyright

We want this book to be used by teachers to help students explore math in a positive way. Feel free to make photocopies, share ideas with parents and colleagues, and use this as a resource draw on. In general, we support this kind of fair use of our materials. Please don't post elements from this book online without citing the source, share large chunks of the book electronically, or sell parts of the book anyone.

Table of Activities

Where to find each activity the FIRST time it's used in this curriculum

Introduction

Welcome to the Math for Love curriculum! We are thrilled to have you on board. We've seen this program make a meaningful difference in the lives of the students who have used it as a summer or supplemental curriculum. We hope it will do so for your students too.

Goals of the Math For Love Curriculum

We wrote this program to be both *play-based* and *rigorous*. The goals of the program are two-fold:

- Improve conceptual understanding of and fluency in mathematics
- Give everyone an opportunity to have fun and enjoy math

Many students haven't had enough time working with conceptual models of mathematics before being pushed into abstraction. To remedy this, the curriculum spends ample time exploring conceptual models, giving students opportunities to work concretely and pictorially while making connections to abstract reasoning.

Program Values

The goals are to strengthen student understanding and deepen their enjoyment of math. The values of the program help work toward those goals:

- Students should play, with both games and ideas.
- Students should have hands-on experiences, exploring math with manipulatives
- Students should experience math as a meaningful, compelling activity, with multiple ways to approach solving a problem, representing a situation, and developing a strategy.
- Students should have time to think deeply about mathematics.

In short, this curriculum is designed to help you build a classroom where students are *doing math* and *thinking math*.

Teacher's Responsibility

As a teacher in the program, you are tasked with establishing a healthy and dynamic classroom environment where these values are expressed. Your responsibilities are:

1. **Engagement**. Create a classroom where your students spend the bulk of their class time actively engaged in mathematical play and problem-solving.
2. **Differentiation**. Help students encounter problems, games, and activities of the right level of difficulty to create engagement.

3. **Thinking**. Get students thinking as soon as possible every day, and help keep them *productively stuck*, actively working to understand, make meaning, and develop ownership of mathematical problems as they think through problems.
4. **Positive Environment**. Help the classroom be a place where students trust themselves, their teacher, and each other, and can make mistakes, ask questions, and grow.

The curriculum is designed to help you in these tasks, and your students and you will get the most out of it if you tackle these responsibilities head on. Here are some concrete ideas on how to go about it.

★ **Be ready with questions**
Rather than simply telling students whether their answers are correct or not, ask them what they did to solve the problem. Ask them what they think the answer is and why. Invite them to share their thinking with you and their classmates. This shows them that you value *their* thinking and contributions, not just the answer.

★ **Model how to play games, and teach how to win and lose**
Students can sometimes get overly attached to winning, and take their wins and losses as deeper signs about themselves. It's best to get ahead of this right away. Talk about how the players of a game are working together to learn about the game, and every loss is a chance to get more information about how to win. Rather than thinking about the other player as your rival, think of them as your collaborator, there to help you learn. You can also adjust many of the games to be collaborative rather than competitive.

★ **Avoid what doesn't involve math; get students into actual, active thinking situations about mathematics as fast as you can**
Our goal is to make the most of classroom time, and avoid things that use up too much time without much gain in mathematical understanding. Start class right away with a Dot Talk or opening game (see the Opener in the daily plan). Use the Math Games and Station Breaks for transitions between Activities. Establish the classroom as a place where we all are committed to working on improving our understanding of math.

★ **Have a *growth mindset* classroom**
Some of your students will believe that they are just bad at math. They will think this is an unchangeable personality trait. The truth is that every student can succeed in mathematics, regardless of how they've done in the past. Convey to your students, early and often, that math is something you *learn* to be good at, not something you just know; how making and learning from mistakes is the key to improving; and how everyone can be good at math if they put in the time and the energy.

★ **Embrace mistakes**
One important way to encourage a growth mindset is to embrace mistakes. They are a natural part of learning, and even more than that, they actually help us learn more

and help us remember what we've already learned. Model for your students how to make mistakes, and how to use mistakes productively. This could be as simple as thanking students when they notice a mistake that you have made.

★ **Give your students *time* to think and explore**
Many students are not given enough time to establish solid conceptual models. Don't feel like you need to rush in order to get through the entire curriculum, if pausing and doing less in more depth would serve your students better. Make sure you don't push students to stop using blocks or pictures too quickly, either. Also note that a central place in the curriculum to practice fluency is in the games. The goal is for the practice and experience of growing mastery to be tied to the experience of playing.

★ **Give your students the right amount of struggle**
We want the students to be 'productively stuck', i.e. we want them to be working on material they haven't mastered yet but not material that is so hard they can't get started. Most of the lessons in the curriculum start easy, so make sure everyone is able to begin, and help students get started on problems with support when necessary. But don't offer so much help that you take away their opportunity to learn. Learning happens when we are trying to do something we know how to begin and don't know how to finish. Keep in mind that many students will be more familiar with the "stuck" part, so try to start them with successes, and then move them slowly toward greater problem-solving stamina.

★ **Value play**
It's easy to feel like students have to suffer to learn math. In fact, the opposite is true. Approach math in a playful way, and you'll see students more willing to struggle and persevere, more willing to take risks and learn from mistakes, and more able to absorb new ideas and put them into practice.

Using this curriculum

If you use this curriculum to supplement math in a classroom, you'll find that you should have enough here to do one or two Math for Love activities a week, some relatively brief, like openers or games, and some activities taking longer. Many of the activities, and especially the games, can be returned to more than once. We recommend you move through the curriculum roughly in order. Use your best judgment, and adapt as necessary.

If you use this curriculum for a summer program, it can serve for a 16-day program of 75 - 90 minute days. If you need it for less time, you can end sooner. If you need something that lasts longer, you'll find many of the activities easily extend to fill a second day. No matter how you use it, we encourage you not to feel like you have to "cover" all the material. Give students the time they need to explore the ideas and activities at a comfortable pace.

Note on materials substitutions

Upscale Pattern Blocks
We recommend Upscale Pattern Blocks (more info at mathforlove.com/upscale) for this curriculum, which include 3 sizes of the 4 basic pattern block shapes. You can use generic pattern blocks (which only come in 1 size) in place of them. If using generic pattern blocks, make sure to remove the orange squares and tan rhombuses before starting the activities in this curriculum. You will be left with the yellow hexagons, red trapezoids, blue rhombuses, and green triangles.

Day Plan

The Day Plan lets you know exactly what's happening on a given day. The components of a typical Day Plan are:

- *Goals*
- *Optional Warm Up*
- *Opener*
- *Activity*
- *Game*
- *Choice Time*
- *Closer*

Goals
These are the learning content goals that are the target of the lessons and activities for the day. These are meant to help the teacher know what to focus on throughout the day. The goals do not need to be shared with students.

Optional Warm Up
Kindergartners get so much out of free play that we recommend starting class with 5 - 10 minutes of free play with the blocks and math manipulatives. The discoveries they make (e.g., 6 pattern block triangles can make a hexagon!) end up being fundamental to the mathematics we'll ask them to do later. If you have the time, it's time well spent. If not, Block Free Play should be a standard option for Choice Time.

Opener
The goal of the Opener is to get students relaxed, focused, and thinking. The teacher typically leads a math talk or game, built to help the students begin thinking and engaging right away. The Openers should be at a level of challenge that provides all students a positive, successful encounter with math first thing.

In general, the Opener should last about 5 - 10 minutes.

Activity, Game, Choice Time
Following the opener, there is a suggestion for an activity, a game, and Choice Time. This is where the bulk of class time will be spent. There are two recommended ways to approach these three elements.

1. Have students rotate between three stations. This is especially recommended when you have additional adults (instructional aides, parent volunteers, tutors) in the room aside from the teacher.
2. Take the whole class through the activities one by one. This is recommended when the teacher is the only adult in the classroom.

Either way you run your classroom, the elements are designed to give students the maximum opportunity to think & engage, practice skills, explore questions, and have fun.

Choice Time includes a suggestion of a small group of past games and activities for the students to try. Block Free Play and Counting Collections can also be regular options for Choice Time activities. This time is a fun and vital opportunity for students to practice skills and explore deeper some of the games they've had a chance to play only briefly when they were formally introduced.

Closer
The Closer is a chance for students to reflect on what they learned or still have questions about in the day, and for the teacher to lead a closing discussion, or pose a final challenge on the new material from the day.

There is a suggested question to pose at the end of each lesson. These are designed to promote reflection some important element of the day's learning. Ideally, these questions will be accessible to everyone, or review. They can usually be discussed in pairs or small groups, and then briefly with the entire class.

Instead, the teacher might prefer to let students discuss another element from the class that they noticed or that they're still wondering about. When students share what they noticed, it's a chance for their observations to come to the attention of the class; when students share what they wonder, it's a chance to see their questions, conjectures, and current state of understanding.

The Closer should take 5 minutes or less.

Other Notes and Best Practices

★ **Math Breaks and Physical Games**
Check out the math-based movement breaks in **Appendix 2**. These are excellent as transitions.

★ **Folder for Worksheets**
Give each student a folder where they can keep their worksheets. If they finish an activity early, they can turn back to their unfinished worksheets and finish them. They can also work on them during Choice Time.

★ **Choice Time**
Provide a structure for Choice Time like putting up the choices on a white board and having students put their names at the games or activities they want to try that day. Ideally, they should both choose the activity that is right for them, and then stick with it for at least half of the time.

★ **Station Transitions**
If you use stations, provide 1-2 minute warnings before station transitions, to apply a gentle transition, cleanup, and—especially at Activity 1—a brief reflection or wrap-up. If you use an alarm, make it a gentle sound (i.e., a gong) rather than an abrasive one (i.e., a clock radio alarm).

★ **Games to send home**
See **Appendix 1** for games to send home. These will help parents/guardians and students play math games at home. You can always send other favorite games home, or encourage students to share games they've learned with people at home. Note that there is no homework for this program otherwise.

★ **Block Free Play**
Giving students 5 - 10 minutes to play freely with the blocks and manipulatives from the class is an excellent way to start every day, if you can spare the time. This will help behavior and focus, and help students develop intuition for the blocks. Block Free Play is always available as a Choice Time option, but consider it as an entry routine as well.

★ **Build Dot Talk images with magnetic ten frames**
For Dot Talks, we recommend you create physical versions of them with magnetic ten frames or other manipulatives. Physical versions are often preferable, since students can manipulate the blocks directly.

Day 1

Goals

1. Establish class norms and community.
2. Play math games and explore situations involving counting and comparing.

Opener

If you have a name game you like to use to get acquainted with your students, feel free to use it! Otherwise try Mingle. (See below, or see Appendix 2.) If the classroom space isn't conducive for Mingle, you can also use Guess My Number (see Day 2).

Activity

Exploration of Materials

Note: you can get a lot of mileage out of letting students explore on their own and make up their own games. There are also extra challenges and guiding questions to help students dig deeper. Those are optional, but may come in handy.

Game

Match the Dots

Choice Time

Today is a good day to practice sticking with a choice. Ask students to choose one of the materials from earlier (Upscale Pattern Blocks, Connecting Cubes, square tiles, dice, Tiny Polka Dot cards, etc.) that they'd like to play with more. Let them sign up for one they'd like to do, and then spend ten minutes with everyone focused on the material or activity they chose. On future days, they'll repeat this routine.

Closer

Today the class focused on playing games and exploring blocks. Ask students what they think those things have to do with learning math? Let students share and discuss.

Some points to emphasize, or bring up if no one else does:

- Math includes learning about a lot of things! Numbers, shapes, and patterns are all part of math.
- You can play to learn math! In fact, it's one of the best ways to do it.

Mingle

The teacher calls out a number (e.g., 3), and the students get themselves into groups of that size as quickly as they can. It might be impossible for everyone to get in a group every time, but each new number gives everyone another chance.

Once students get in groups, they can learn each other's names.

In the basic game, just call out single numbers. Once students get the gist, you can call out addition or subtraction problems (i.e., "get into groups of 7-4"). Don't forget to call out a group of 1 and a group of however many students are in the entire class at some point in the game.

Tips for the Classroom

1. The teacher can get into or out of the groups in order to make sure no student is by themselves.

Exploration of Materials

Students are encourage to explore and play with the materials they'll be using in the class this summer. Free play is ideal, as long as the students stay relatively focused.

If you need to focus them further, consider using these challenges to motivate students who need extra guidance. Make sure ten frames are available for students who want to count tiles, Connecting Cubes, or counters.

Pattern Block Building Challenges

Pattern Blocks Building Challenges: first, students take option 1, and free build. For students who need additional challenges, look to the next two options.

- **Option 1**: Free build with Upscale Pattern Blocks.
- **Option 2**: Cut out the 12 cards on the following page, and separate into two piles. For a basic building challenge, a student picks a card at random, and then builds according to what is on the card.

Example 1: Build a *Person*.
Example 2: Build something with *12 blocks*.

- **Option 3**: Advanced challenge: a student gets two cards, one which tells them what to build, and the other how many blocks to use.

Example: Build a *Triangle* using *15 blocks*.

Piles of Tiles

with the color tiles and/or the Connecting Cubes

Let students free play and build with the tiles and Connecting Cubes. For students who need more direction, give them a pile of tiles, and challenge them to figure out which color occurs most.

Challenge Prompt: In this pile of tiles, which color is there more of? Write down your guess, and then find out.

Tiny Polka Dot Games and Challenges

with the Tiny Polka Dot decks

Challenge 1: Pick a suit (i.e., a color). Arrange the Tiny Polka Dot cards in that suit from smallest to biggest. What will it look like if you organize an entire deck?

Challenge 2: Pick a suit. Count all the Tiny Polka Dot cards of that color (in one deck).

Challenge 3: Pick a suit. Count all the dots in that suit!

Two-Color Counters Challenge

Challenge Prompt: Shake a handful of counters and then spill them out. Without counting, are there more black counters or white counters?

Once you've guessed, count the counters of each color and see which has more.

Try again with more counters!

Dice

Challenge Prompt: pick up six dice. If you roll them all, which number will get rolled the most? Make your prediction, then try and see.

Pattern Block Building Challenges

Flower	Bird	Person
Pattern	Triangle	House
5 blocks	8 blocks	10 blocks
13 blocks	15 blocks	20 blocks

Piles of Tiles

Which color tile occurs the most in this pile?

Write your name under the color you think has the most.

Red	Green	Blue	Yellow

When you have voted, count how many of each color there are.

Match the Dots

Topics: Counting and cardinality, subitizing, recognizing numbers
Materials: Tiny Polka Dot cards
Common Core: K.CC.4, K.CC.5, MP6, MP7

The numbers look different. Which ones are the same?

Why We Love Match the Dots

This simple game of counting and matching helps students see how different-looking collections of dots may actually represent the same number.

The Launch

Choose a student volunteer to demonstrate the game, or put the cards where everyone can see them (center of a circle, document camera) and play with the entire class, where you find a match, and then the students raise their hands if they see a match.

The rules are simple: deal a collection of cards face up. On your turn, simply find two cards of the same number, and remove that pair from the board. There aren't really winners or losers in this game—you just play taking turns until all the cards are gone. For early games, pick just two suits at a time, and start with numbers the students know. For a first game, you might pick Teal 0-4 and Blue 0-4. In subsequent games, students can play with larger numbers and additional suits.

Prompts and Questions

- How do you know those two cards have the same number of dots?
- I'm going to pick this card. It has two dots. Do you see a card that matches it?
- Are you sure those two are the same? Let's count them together.

The Wrap

Ask students which cards were easiest and hardest to count.

Tips for the Classroom

1. Start students with very easy numbers, and slowly let them increase the difficulty. Every step up should feel like a fun new challenge. "Do you think you're ready to add in some sixes?"
2. The game is more fun if you're using an even number of suits, so every card gets taken in the end.
3. This game evolves into Dot Memory and Dot Fives. If students are ready for a greater challenge, you can show them those games.
4. If students aren't ready for Match the Dot, you can let them play Hungry Numbers, or just play with the cards, or organize them into color piles.

Day 2

Goals

1. Practice counting with Counting Collections routine.
2. Play games involving numbers, number order, and counting.

Warm Up (optional)

Block Free Play

If you have time as students are entering the classroom, it's highly recommended that you make blocks available at desks, or at stations, and let students explore the materials and chat with each other for the first 5 - 10 minutes of class. This immediately establishes the classroom as a place where students engage with materials and each other. Consider this as something to do every day.

Opener

Guess My Number

Activity

Counting Collections

Game

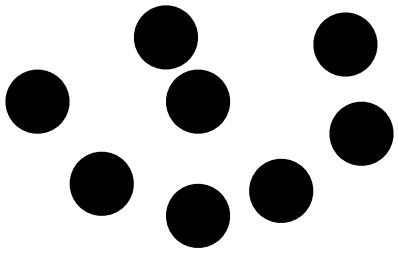
1-2 Nim

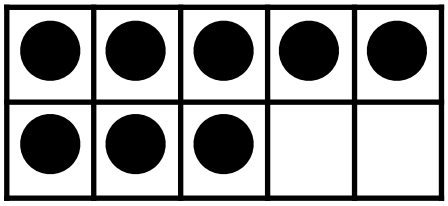
Choice Time

Block Free Play
1-2 Nim
Match the Dots
Counting Collections

Closer

Ask students what kind of arrangements were easiest to count in Counting Collections, and which were hardest. If you have magnetic dot ten frames, place a jumble of 8 dots on the board, and ask students to count them. Then arrange the dots in a ten frame with three outside it. Is that easier to count? What strategies might students use next time they do Counting Collections? (You can draw these otherwise. Alternatively, you can use Tiny Polka Dot cards from the green, unordered suit, vs. the blue, ten frame suit)





Guess My Number

Topics: Greater than/less than, logic
Materials: Whiteboard or paper and pencil
Common Core: MP1, MP3, K.CC.C7

Guess My Number is a quick, fun, and easy opening game to lead with a small group or the entire class. Students try to guess the number you're thinking of in the fewest number of guesses possible.

Why We Love Guess My Number

It's hard to think of a game more natural than Guess My Number. Kids can immediately start playing, and have a great time doing it. The game is also a perfect way to introduce the basics of logical thought and strategy, and let the game do the teaching with only minimal extra observations from the teacher. Surprisingly replayable and enjoyable, this is a great game to play early and often.

The Launch

As you write the numbers from 1 to 10 down on the board, tell your students that you are going to think of a number from 1 to 10, and they will try to guess it in the fewest number of guesses possible. After every guess, you will tell them whether your number is greater or less than their guess.

Example
Teacher: Who would like to make the first guess? [Students raise hands. The teacher calls on different students for each guess.]
Student: 3.
Teacher: My number is not 3. But my number is greater than 3. [Optional: write "My number >3".] Are there any numbers I can cross off my list?
Student: It's not 3. [Teacher crosses off the 3]
Student: It's not 2 or 1 either.
Teacher: Right. Because my number is greater than 3, but those numbers are less than 3 [Teacher crosses off 1 and 2.] Who has another guess?
Student: 9.
Teacher: My number is not 9, but my number is less than 9. [optional: write "My number <9".] Can I cross any more numbers off the list?
Student: The 9 and the 10.
Teacher: Because my number is less than 9, so it can't be 9 or 10. [Crosses them off. Looks at the board.] So the only options left are 4, 5, 6, 7, or 8. Take a minute to think about what would be a good next number to guess. Then tell someone sitting next to you what you would guess next, and why. [Students pair and share.] Who has another guess?

Student: Is it 7?
Teacher: My number is not 7, but my number is less than 7 [optional: write "My number <7]. What numbers can I cross off?
Student: 7 and 8.
Teacher: Right. Because my number is less than 7, so it can't be 7 or 8. [Crosses them off.] So the only options are 4, 5, or 6. Who has another guess?
Student: Is it 5?
Teacher: My number is not 5, but it is… greater than 5. [Students' hands shoot up.] Whoever thinks they know my number, say it together.
Students: 6!
Teacher: You got it! Now that took you [counts] 1, 2, 3, 4, 5 guesses. Who thinks they could do it in fewer? [Optional: play again.]

Prompts and Questions

- How can you guess my number in the fastest, most efficient way possible?
- Talk to a neighbor about what you think the next guess should be, and why.
- How many numbers do you think that guess will cross out?

Tips for the Classroom

1. **Cheat**! By which I mean, don't actually choose your number ahead of time. If students guess 1, tell them your number is more than 1. Always make each guess give them the least amount of information possible, and deny them the lucky guess. Make them work for it, and they'll be more invested in working smarter.

   AVOID:
   Student: Is it 8?
   Teacher: It is! Lucky guess! You got it on the first try.

   BETTER:
   Student: Is it 8?
   Teacher: My number is not 8, but my number is less than 8.

2. You can slowly expand up to larger ranges of numbers as students are ready for them. I'll usually go up to 12 after a few games, and soon to 20. Kids love to see the game get harder, as long as it doesn't get too hard too fast. And because you write all the numbers up on the board at the start, they can always see what needs to happen.
3. If kids make a bad guess, don't try to steer them toward a good guess right away. But you can ask the students after you write the guesses down which guesses were most helpful, or whether they would make a different guess if they could take it back.
4. Don't play for too long at one time. One or two games is usually enough to get the kids mentally alert and ready for whatever is coming next.

Counting Collections

Topics: Counting, skip counting, addition
Materials: Paper, pencil, objects of many types (button, beans, stones, pencils, markers, blocks, etc.), ten frames
Common Core: K.CC.A.1, K.CC.A.2, K.CC.A.3, K.CC.B.4, K.CC.B.5, K.MD.B.3, MP1, MP6

Kids love to count things. This is a simple exercise, but a great way to get kids excited about arithmetic. Counting Collections also seeds ideas about how the place value system helps make counting work.

Why We Love Counting Collections

So much of the work of teaching math in elementary school is about sharing arguments that make counting faster and more efficient. But why should the students care if they don't need to count anything? By making counting explicit, hands-on, and fun, this activity actually provides a motivation for the mathematics of place value (counting by ones, tens, and hundreds), addition (counting two or more smaller groups and adding them together), and multiplication (skip counting, arranging objects in arrays). Not only that, each new method and algorithm can be immediately tested in a concrete setting. Counting Collections are an indispensable tool for young children.

The Launch

Set collections of objects in different places in the room. Examples might include cups of legos, straws, blocks, pencils, buttons, markers, etc. Make sure that there are enough collections with a small numbers of objects (i.e., 6 - 20), as well as some with larger numbers.

Point out the collections around the room. Let students know that their job will be to take an inventory of the objects in the room—that is, to count how many objects are in each collection. In addition to recording how many objects are in a collection, they can make a sketch to remember how they got the answer.

Students will work in pairs. Each pair will get to choose a collection and count it. For each collection, they will record how many they counted on a sheet of paper next to the collection. Multiple groups will write the number they think is in the collection. There may be different numbers! If so, this is worth revisiting in the wrap up.

Main Activity

The students count and record their numbers.

The teacher can use this time in many ways. She can observe how students are counting, and take notes on the strategies they're using and where they are in their developing

understanding of numbers. She might distribute ten frames, rubber bands, cups, or other devices to help kids count or bundle objects.

Prompts and Questions

- What strategies are you using to count?
- What number do you find is easiest to count by?
- Can you tell how your partner is counting by looking at their picture?

Wrap Up

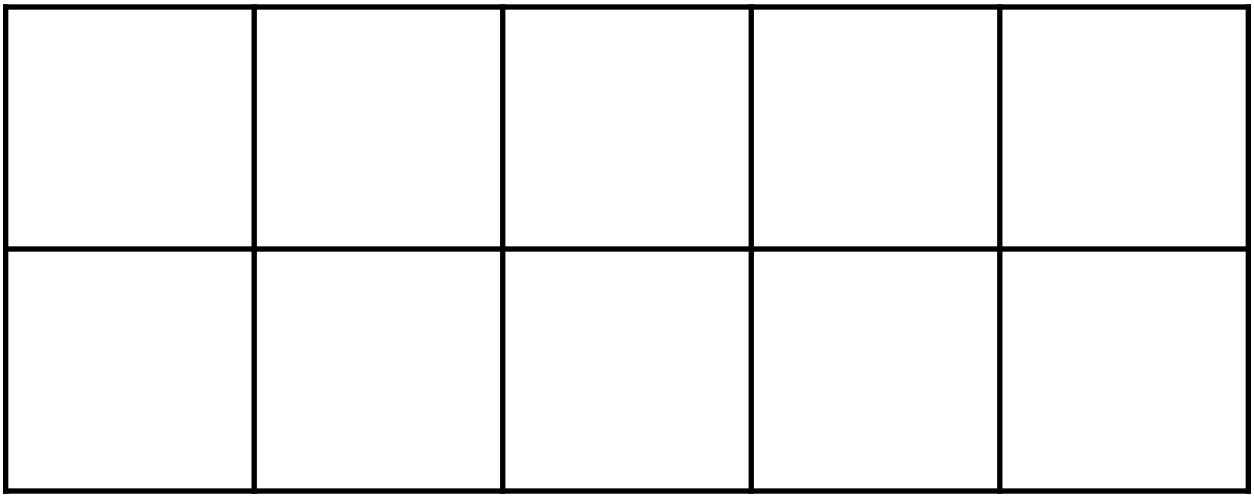
You don't have to wrap up this activity after the first day; Counting Collections can be something to return to, to let students test their methods on larger and more difficult groupings of objects.

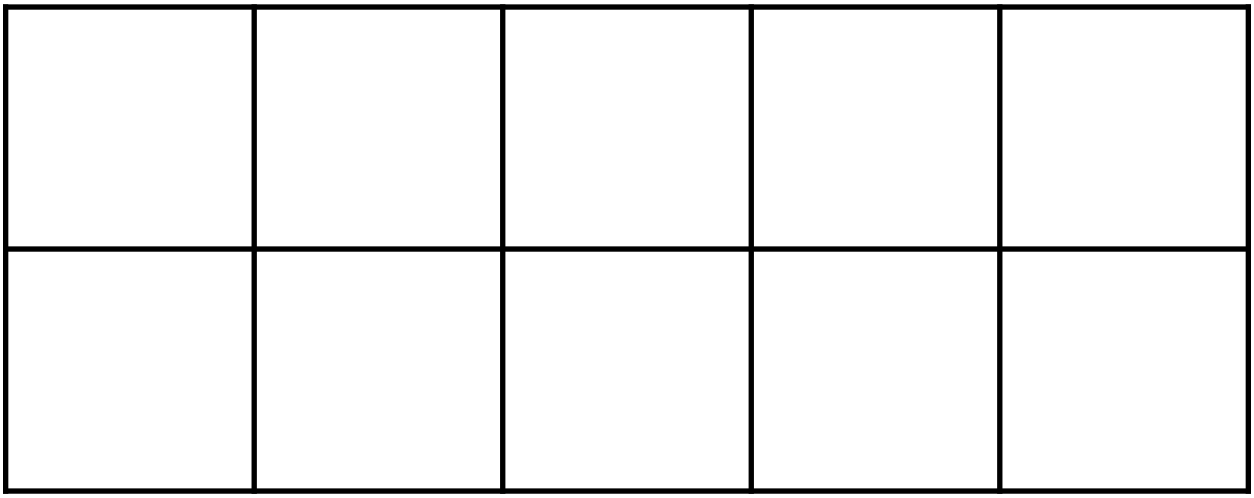
However, it can be nice to discuss different strategies along the way. After students have done their counting for the day, ask for reports on counts, and discuss strategies kids used for counting. Was it easier to count by 2s? By 10s? Was it helpful to arrange the objects in some neater organization, like a grid, or on top of a ten frame? What other strategies did people have? Did all the groups who counted the same thing get the same answer? Which counting methods are most accurate? Which are easiest?

Tips for the Classroom

1. You can differentiate Counting Collections most easily by including collections with many or fewer objects to count.
2. On the first day of Counting Collections, make sure you have many small collections (5 - 20 objects) for students to count.
3. Make ten frames available as a counting aid.

Ten Frames





1-2 Nim

Topics: Logic, patterns, addition, counting, subtraction
Materials: Counters (tiles, beans, pennies, etc.) and/or paper and pencil
Common Core: K.CC.A.2, K.CC.B.4, K.CC.B.5, K.OA.A.1, K.OA.A.2, K.OA.A.5, MP1, MP2, MP3, MP5, MP7, MP8

You can take one or two counters from the pile. How do you get the last one?

Why We Love 1-2 Nim

Nim is fun, challenging, and rewarding for a wide range of kids. Done right, it can engage everyone from Kindergarten to upper elementary kids, and connect to basic counting and arithmetic up to division. Completely unlocking the game is an exciting and powerful achievement for a student.

The Launch

Take a volunteer to play a demonstration version of the game. You can explain the game to them very quickly and get to playing (which is how the other students will learn the game, as they watch). It's a good idea to get students to suggest moves to you as you play—quietly, by holding up one or two fingers. Ideally, you should win the initial games. See the video at mathforlove.com/video/rich-task-1-2-nim-lesson-plan-with-dan-finkel for more ideas.

How to Play

Nim is a two-player game. You start with a pile of 10 counters. On your turn, remove one or two counters from the pile. You must take at least one counter on your turn, but you may not take more than two. Whoever takes the last counter is the winner.

Example Game

Player 1 takes one counter, leaving 9.
Player 2 takes one counter, leaving 8
Player 1 takes one counter, leaving 7.
Player 2 takes two counters, leaving 5.
Player 1 takes one counter, leaving 4.
Player 2 takes one counter, leaving 3.
Player 1 takes one counter, leaving 2.
Player 2 takes two counters, leaving 0 and winning the game.

The Work

Playing in trios is a nice structure here. Two players can play while the third observes. Then they switch roles. That prevents a pairing from feeling stale after a few minutes.

The central question here is: how can you win 1-2 Nim? Encourage students to understand their losses as opportunities to improve, or gain new strategies for the future. For Kindergartners, even if they don't get into deeper strategic elements of the game, there's a huge benefit in tracking how the pile shrinks as they take one or two counters away from it. Naming the numbers as they go can help emphasize this. Also consider having them play on ten frames.

A key idea is that you can play with fewer counters in the pile, and that actually makes it easier to think about what to do. Gathering the students and challenging them to games with fewer counters (they can request how many fewer) is a nice way to reengage everyone.

Prompts and Questions

- What move should I (the teacher) make?
- How did you/they/I win that game?
- What do you think your/my opponent will do if you/I take two counters?
- Would you like to take back your move?
- What have you noticed about this game?

The Wrap

Students will likely be in different places with respect to their strategy by the end of the lesson. You can close by asking for another challenger to play you and see if they can beat you. If they can, the class will be elated. If they can't, that means there's still more for them to figure out before they become Nim Masters.

Tips for the Classroom

1. Demonstrate the game with volunteers for at least three games (or many more!), until you are certain everyone understands it and is excited to play.

2. When demonstrating 1-2 Nim, narrate the game out loud, using mathematical language, and leaving empty space for students to chime in: "My opponent just took 2 leaving... [wait for students] 5 in the pile. Who has advice for what I should do next?"

3. Remind students that they will lose many games as they play, and that every loss is an opportunity to learn. Can they steal the strategy of the person who just beat them? Point out how students are trying out new strategies as they play you in demonstration games.

4. As kids play each other, circulate to see what strategies they are developing. Challenge them to play you, and see if they can beat you.

5. Encourage student conjectures, but do not call them as true or false. Challenge students to break their own conjectures.

6. This game is great for station work once students already know how to play it.

7. We use the term “the 3 trap” to describe what happens when you give your opponent a pile of three counters. Understanding how to win boils down to understanding what pile sizes you want to leave your opponent with.

8. There are two incredibly powerful approaches to solving Nim. We’ll discuss them here, BUT don’t be in a rush to push the kids to find them. Help students only as far as they seem ready to go, and if for many weeks, or even the entire summer session, they never articulate a correct answer to the question of how to win at Nim, they’ll still get lots of beneficial mathematical practice by simply playing the game.
   The first approach is to simplify. How could the game be easier? What if the pile had only one counter? From this place of almost absurd simplicity, we slowly raise the difficulty. What about two counters? Three counters?

   The second approach is to organize the data in a coherent way. A table does this very nicely.

Number of Counters	Winning Strategy
1	Go first. Take 1.
2	Go first. Take 2.
3	Go second.
4	?
5	?

9. We discourage a three-player game. Normally trying out different numbers of players is a great impulse. In Nim, it leads to spoilers, who can’t win, but can choose who does win.

10. Optional Homework: teach 1-2 Nim to a friend or family member.

Day 3

Goals

1. Practice counting and building shapes from smaller shapes.
2. Play games involving numbers, number order, and counting.

Warm Up (optional)

Block Free Play

Opener

Dot Talk - see Appendix 3 for Dot Talk images

Activity

Pattern Block Fill-Ins 1

Game

Back and Forth

Choice Time

Block Free Play
Back and Forth
1-2 Nim
Match the Dots
Counting Collections

Closer

Hold up two hexagons, and ask students how many red trapezoids they think it would take to cover them. Give them a minute to think on their own, build with their own blocks, and defend their ideas to a partner. Then discuss as a class and confirm the answer (Answer: 4). Then ask them how many blue rhombuses they think it would take to cover the same two hexagons. (Answer: 6)

If time permits, you can repeat with green triangles (Answer: 12).

Dot Talks

Topics: Mental math, numerical fluency; argument & critique
Materials: White board & projector
Common Core: K.CC.2, K.CC.4, K.CC.5, K.OA.2, K.OA.4, K.OA.5, K.NBT.1, and especially MP3

This mental math routine creates powerful positive habits for students.

Why We Love Dot Talks

In just 5 - 10 minutes, these openers get all students involved, help strengthen fluency, intuition, and mental math strategies, improve students' ability to explain and critique solutions, and allow teachers a valuable window into their students' thinking.

The Launch

The talk starts very simply. The teacher projects the dot image on the board where all students can see it, and asks students to figure out how many dots there are.

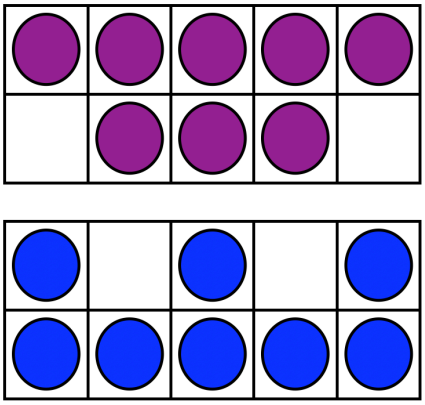
A common approach is to give students some time to think about the questions on their own first, and then share with a partner. Once students have had time to think the question through, lead a class discussion where students share their answers and approaches.

The key elements to these talks are a de-emphasis on speed and right answers and an added emphasis on process and communication. Expect some disagreements over the answers, and try to use those disagreements as a motivation for students to articulate their ideas to their classmates.

The Work

Students may have all kinds of approaches to count the dots, from counting by ones, twos, fives, or tens, to using the ten frames, to using the colors of the dots. Their conversations and arguments are the key, and the more you communicate that you're most interested in understanding what they're thinking (and writing it down so others can follow), the more they'll step up with more ideas to share.

In the image to the right, for example, one student might see "8 + 8," while another sees that if one dot is removed, the remaining make 3 rows of 5, meaning the total is one more than 15. Another students might imagine moving 2 dots from the bottom frame to the top to complete a set of ten, and thus see 16 total. Another might notice that the total is 4 less than 20.



Prompts and Questions

- Who would like to defend this answer?
- I don't quite follow. Do you mean I should count this group first?
- How did you do that/know that?
- Does anyone else think they can explain what Shawn is saying?
- Turn to the person next to you and explain how you counted.

Tips for the Classroom

1. Students will be looking to see if you indicate what the right answer is. Don't favor right answers over wrong ones. Make sure that the explanations are what matters.
2. A smart protocol is to have students put a thumb up at their chest rather than waving a hand in the in the air.
3. Give students constructive language to use in the discussion, like, "I respectfully disagree, because…" and "I agree with _____, because…"
4. Always keep the environment safe and positive.
5. Don't worry if you don't reach total consensus on every problem. Sometimes a student will need more time to process. You can move on when it feels like it is time.
6. Doing short (5 - 10 minute) Dot Talks regularly is more powerful than long ones infrequently. Do a maximum of two talks per day.
7. Find Dot Talk images in Appendix 3.

Pattern Block Fill-Ins 1

Topics: Shapes, counting, addition
Materials: Upscale Pattern Blocks, fill-ins, pencil and paper
Common Core: K.CC.4, K.CC.5, K.OA.1, K.G.6

Note: it is possible to use generic pattern blocks instead of Upscale Pattern Blocks. If doing so, be sure to remove the orange squares and tan rhombuses before starting this activity, or tell students not to use them.

How many blocks does it take to fill in the shape?

Why We Love Pattern Block Fill-Ins 1

This activity differentiates beautifully, since anyone can begin filling in shapes with blocks, and because there are so many additional challenges that students can explore beyond the initial one. Counting the blocks can be quite challenging, since students need to keep good track of where they started and which blocks they've already counted.

The Launch

Show one of the fill-ins and take guesses on how many blocks it will take to fill in a hexagon. Then fill it in, and demonstrate how to count the blocks and write in how many you used. Ask whether you should try to use fewer blocks or more blocks when you fill it in a second time.

Prompts and Questions

- How many blocks did you use to fill in that shape?
- Show me how you counted. What if you grouped by fives/tens?
- Do you think you could fill it in with more/less than last time?
- What's the smallest number of blocks you could use?
- What's the greatest number of blocks you could use?

The Wrap

Discuss which numbers of blocks students used to fill in the 3-hexagon shape. What was the largest number students used? What was the smallest? Could they have done even more or less?

Tips for the Classroom

1. For some students, just filling in the shapes without gaps may be a challenge.
2. You can use blocks themselves instead of paper. Have students use three small hexagons and cover them up with other blocks. You can also substitute a medium or large Upscale

An Upscale Pattern Block fill-in using 5 blocks

Pattern Block, so students use just a single block instead of a group of three (that might shift around as they build).

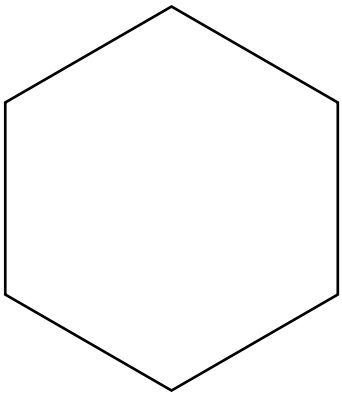
3. Some counting methods to encourage and emphasize:
   A. Count how many of each shape first, then add.
   B. Gather shapes into groups of ten to make it easier to track counting.
   C. Place shapes in a straight line or in a ten frame to organize the counting.

Name_____

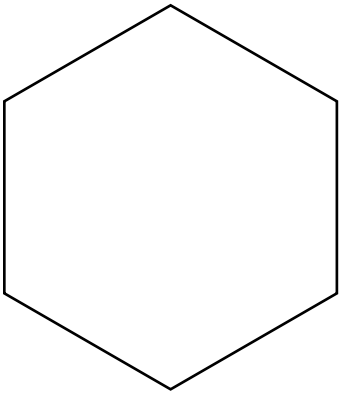
Pattern Block Fill-ins

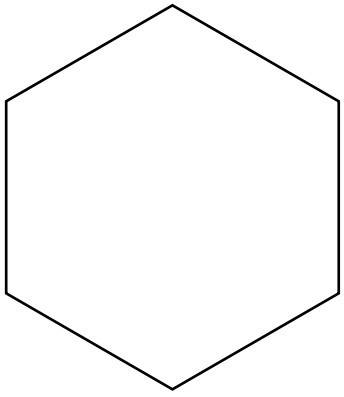
One fun thing about pattern blocks is that there are lots of way to fill in shapes.

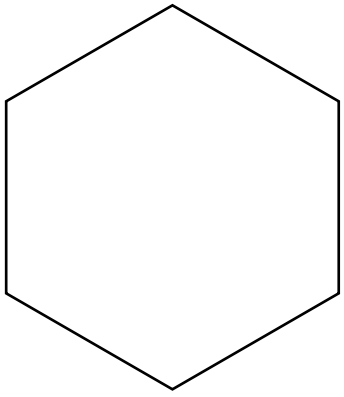
You could fill this shape in with a yellow hexagon. But you could also do it in other ways.



Fill in the hexagons with pattern blocks in different ways.

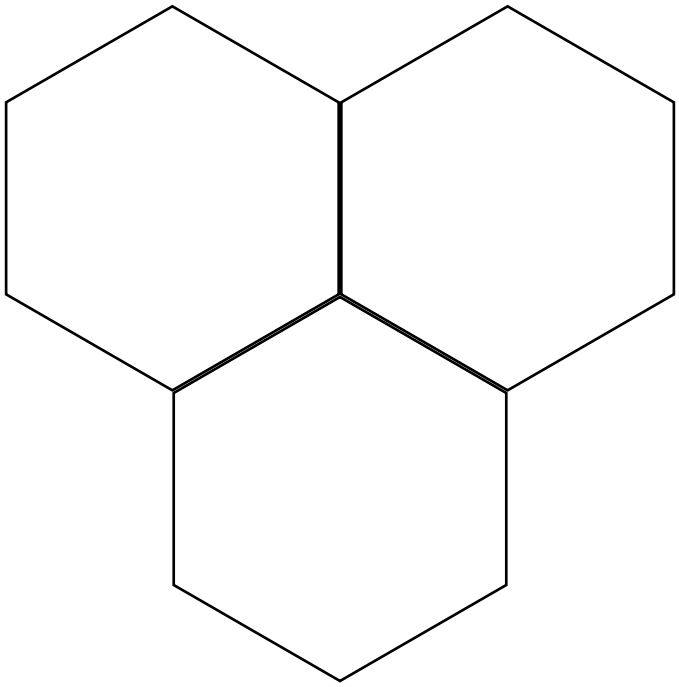






Name_____

Now try filling in this bigger shape. You could use 3 hexagons. But how else could you do it?

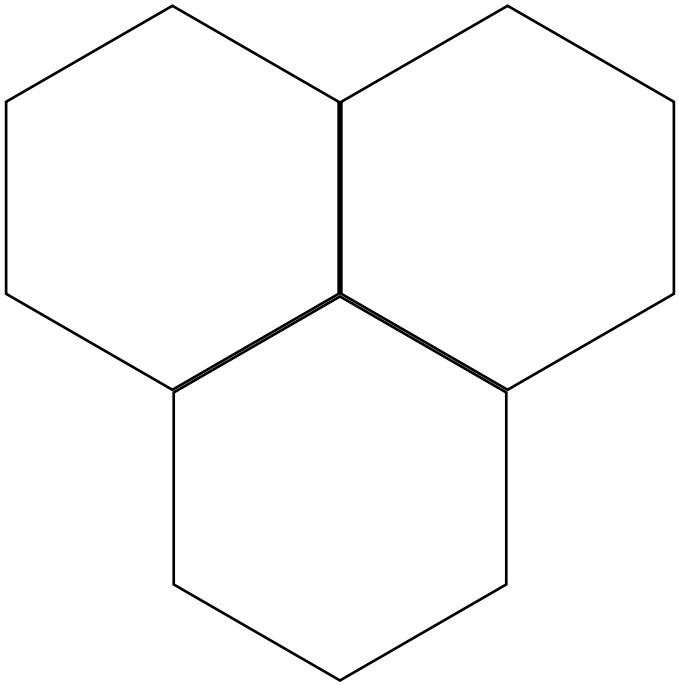


How many blocks did you use?

I used _____ blocks

Name_____

Now try again, and use even more blocks!



How many blocks did you use?

I used _____ blocks

Back and Forth

Topics: Counting, the number line, addition, subtraction
Materials: Game board, counters, die
Common Core: K.CC.2, K.CC.4, MP1

How can you pick up the counters?

Why We Love Back and Forth

This simple game involves just enough choice to engage students, while making connections between counting, the number line, addition, and subtraction.

The Launch

Demonstrate playing the game with a student. The rules are simple: place three counters on the board in any three spots, from 0 to 10. Then roll the die and move your "pawn" (a counter of a different color) forward or backward until you collect all three counters by landing on them. See the example game with rules on the next page.

After each roll, ask the student whether they'd like to add (go higher/go to the right) or subtract (go lower/go to the left). You can also pause before rolling to ask the class what number would be good to roll.

Note that this game can be played independently/collaboratively (use one pawn and collect all the counters) or competitively (use two pawn; whoever gets the most counters wins). Choose the version that's best for your students, or let them choose their preference.

Prompts and Questions

- What number do you wish you'd roll right now? What would you do if you rolled it?
- Are you going to go forward, or backward?
- Show me how you count forward 4.
- Show me how you count backward 3.
- Can you predict where you're going to end up without counting?
- Why do you think it's a good move to go backward right now?

The Wrap

While this game doesn't require a wrap-up conversation, there is an interesting question of what happens if you go off the board. (This can be forced if a pawn is at 5, and the player rolls a 6.) If we were going to extend the board, what would be put past 10? What would be put before 0? This can be a practical or fanciful discussion.

Tips for the Classroom

1. Note: the instructions are on the game board in case you send the board home to parents to play. Don't expect students to learn the game by reading the directions.
2. Using foam dice can help to minimize classroom noise.

Back and Forth

Goal:

Move back and forth until you collect all the counters!

How to Play:

Choose 3 counters that are the same color.
Place them on the board, on whatever numbers you want. For example, I put counters on 2, 7, and 8.

0	1	2●	3	4	5	6	7●	8●	9	10

Pick a different color counter to be you.
Start at 0.

0○	1	2●	3	4	5	6	7●	8●	9	10

On your turn, roll the die.
Move **forward** or **backward** by the number you roll.
You choose, but make sure you don't go off the path!

When you land on a space with a counter, pick it up!
You win when you have collected all your counters!

Example

I put counters down on 2, 7, and 8. I Start at 0.

Turn 1

0○	1	2●	3	4	5	6	7●	8●	9	10

I rolled a 3. I moved forward 3 spaces.

0	1	2●	3○	4	5	6	7●	8●	9	10

Turn 2

I rolled a 6.
I decided to add, and went forward 6 spaces to 9.

0	1	2●	3	4	5	6	7●	8●	9○	10

Turn 3

I rolled a 1. I decided to subtract, and went back 1 space to 8.

0	1	2●	3	4	5	6	7●	8● ○	9	10

There is a counter at 8, and I picked it up.
Only two more to go!

Back and Forth

How to Play:

Put 3 counters on the board.

Start at 0. On your turn, roll the die.

Move **forward** or **backward** by the number you roll.

Try to collect all the counters!

0	1	2	3	4	5	6	7	8	9	10

Day 4

Goals

1. Practice sorting and counting.
2. Play games involving matching numerals to numbers of objects in various arrangements.

Warm Up (optional)

Block Free Play

Opener

Same but Different

Activity

Sort and Count

Game

Dot Memory

Note: Dot Memory is exactly like Match the Dots, except that instead of playing with the cards face up, we deal them face down. Take turns turning up two cards, and keep them if you turn up two that are the same number.

Choice Time

Block Free Play
Back and Forth
Match the Dots or Dot Memory
Counting Collections

Closer

Sometimes we see a friend do something that gives us a good idea or helps us learn. Ask students to share some thinking another student did that helped them understand or learn something today.

Same but Different

Topics: Geometry, addition, counting, estimation, symmetry, equality
Materials: Blocks of all kinds, ten frames, counters, pencil and paper
Common Core: K.CC.5, K.OA.1, K.OA.2, K.OA.3, MP1, MP3, MP4

What's the same? What's different?

Why We Love Same but Different

This wonderful opener (developed by educator Sue Looney), is amazing for helping students reason, observe, discuss, and disentangle understandings of sameness and difference that end up being key to higher level mathematical thinking later on.

The Launch

The teacher shows an image that consists of two distinct parts, and asks: what's the same? What's different?

Students then discuss what they see, first in pairs or small groups, and then with the whole class. Once students have shared 3 or 4 attributes that are either the same or different, you can wrap up the opener, or try a second image if you have time.

Here are some attributes to consider when discussing images:

- **Material** - are both images made of the same stuff or not?
- **Number** - are there the same number of things on both sides or not?
- **Shape/arrangement** - are the things arranged in a line, in a circle, etc.?
- **Grouping** - are they grouped in twos, in threes, in fives, etc.?
- **Color**
- **Orientation** - are they pointing up, large side down, etc.?

Ideally, students will find other attributes too!

Consider the image to the right, for example. Students might notice that:

- Both images have pine cones - that's the same.
- But the number of cones is different: one has 6 and the other has 8.
- The shape of the pine cones is also different: they're in a circle in the first image, and not in the second picture.
- Maybe we could argue that the pine cones are arranged in three lines in each image though. Should that count as the same? Students may agree or disagree here!

Prompts and Questions

The teacher can be very active in helping the students to see different possibilities for number representations if they're having trouble getting started.

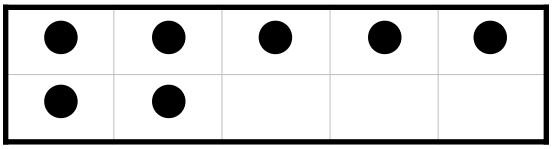
- I see they are the same color. What is something that's different about them?
- We've heard two things that are different. What's something that's the same?
- We just heard someone argue that there are a different number in these two images. Turn to your partner and figure out which image has more in it.

The Wrap

The main idea with this lesson is that there are lots of ways to make different numbers. To close, discuss some of the ways that may have been new or surprising to students.

Tips for the Classroom

1. The teacher can participate in this activity too, and create representations that might not come up for the students. Here are a couple of important ones to slip in if they don't come from students:
   a) Addition: use two colors of counter to make a number, and describe the representation as addition, i.e., 1 green + 2 red = 3 tiles.
   b) Subtraction: Use negative space on a ten frame to represent a number. For example, to represent 3, you could fill in seven spots on a ten-frame, and ask if anyone sees how this represents the number 3.



2. If students need a little more time, you can be lenient about the minute time limit. If students are done, you don't necessarily need to wait the full minute. If students are spending too much time making overly detailed drawings, keep the limit strict, and encourage them to simplify their drawings.
3. For an advanced version of this game, you can actually make simple addition or subtraction expressions for students to build/draw/model. For example, 1 + 2, or 4 - 1.
4. Students who finish quickly can make multiple representations of a number.

Sort and Count

Topics: Counting, skip counting, addition, multiplication (optional)
Materials: Paper, pencil, objects of many types (button, beans, stones, pencils, markers, blocks, etc.), ten frames (recommended)
Common Core: K.CC. 1, K.CC.2, K.CC.3, K.CC.4, K.CC.5, K.MD.3, MP1, MP6

First sort. Then count. While pile has the most?

Why We Love Sort and Count

Sort and Count is Counting Collections with an extra twist. This is a great way to incorporate sorting, counting, and comparing, and pave the way to adding and subtracting.

The Launch

Set collections of objects in different places in the room. The objects should be various colors and sizes, in some regular way. Square tiles, connecting cubes, or Upscale Pattern Blocks are a perfect example, since they come in different colors, but match otherwise.

Let the students know that their job will be to **sort** each collection into groups by color or type (in the natural way, depending on what objects you're using). Then they'll **count** how many there are in each group and write those numbers down. Finally, they'll **compare** the piles and arrange them from the smallest to the largest, and, if they're ready, **write** the numbers in order from least to greatest, optionally using a < sign between them.

Prompts and Questions

- How do you know the red pile has more than the blue pile?
- How many more does it have?
- Try writing those numbers down. How many in the shortest pile?
- Have you tried using the ten frame to help you count? I bet it would be helpful. Let's use it.
- (extension) How many tiles are in the green and blue piles altogether?

The Wrap

You don't have to wrap up this activity after the first day; like Counting Collections, Sort and Count can be something to return to, to let students test their methods on larger and more difficult groups of objects.

However, it can be nice to discuss different strategies along the way. After students have done their counting for the day, ask for reports on counts, and discuss strategies kids used for counting. Was it easier to count by 2s or 1s? Was it helpful to arrange the objects in some neater organization, like a grid, or on top of a ten frame? What about

comparing the piles? Was there an easy way to tell if the numbers/piles of objects were bigger or smaller than others?

Tips for the Classroom

1. You can differentiate Sort and Count most easily by including collections with many or fewer objects to count.
2. On the first day of Sort and Count, make sure you have many small collections (5-20 objects) for students to count.
3. Make ten frames available as a counting aid.
4. It's very natural to extend Sort and Count into questions of addition and subtraction. For addition questions, ask how many objects are in 2 or more piles. (How many tiles are in the green and blue piles altogether? How many tiles are in the red and yellow pile? What about all the piles?)
5. For subtraction, ask how many more are in the biggest than the second biggest. (There are more yellow tiles than red, huh? How many more?)