Square Building

Materials and Prep
Square tiles, graph paper, and pencil.

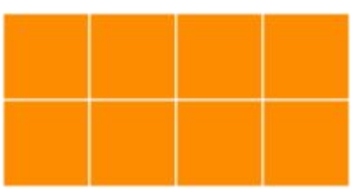
Motivating Question
Is there a pattern in the number of square tiles it takes to build a larger square?

Launch
To launch the lesson, build a few squares out of square tiles, count how many tiles you need to make each one, and record those numbers.

Build a square using just one tile. Ask the students how many tiles would be needed to build the next largest square. Have students think about it and share with the person next to them. Many students will probably predict that 4 square tiles will work to build a square. Put four tiles together to form a square.

Ask the students how many tiles would be needed to build the next largest square. Have students imagine it, then share with the person next to them. Some possible answers shared by students may include 8, 12, 16, and 9 squares.

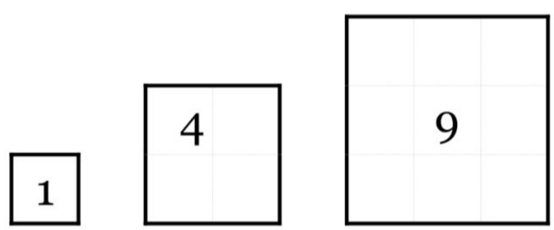
Explore the idea of using 8 squares. Build the 2 by 4 rectangle. The students will notice that it is a rectangle, not a square. Ask the students if anyone can explain why this is not a square. What needs to be true about squares?



A student may suggest that all the sides have to be the same length. Looking at the rectangle the students notice that the vertical side is 2 edges long, and the horizontal side is 4 edges long. Different side lengths means it is a rectangle, not a square.

Launch Key Points

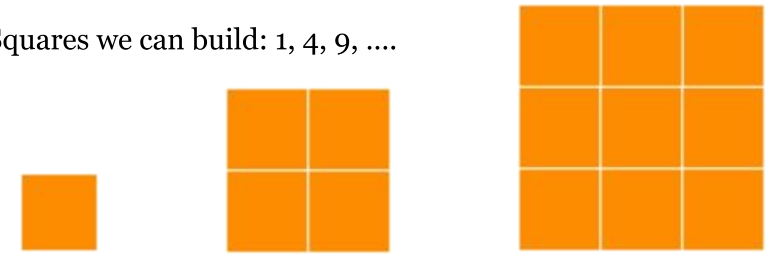
- Make sure students know how to check if they've built a square by looking at side length.
- The launch begins with concrete counting and building. Students will make the connections to multiplication and arrays as they work, and in the Closer.



Think aloud about moving the squares around so that the tiles make a 3 by 3 square missing a corner. Students will see that it needs one more tile.

At this point note that this looks like a pattern starting, and write it down where the students can see it.

Squares we can build: 1, 4, 9, ...



Explain to students that their job today will be to find the next numbers in the sequence by building larger squares. Remind them to make sure that their squares are actually squares! All the sides need to be the same length. Encourage them to try to make their list complete by finding the next largest square in order.

Work

Give students 15-20 minutes to build squares of different sizes and write down their list of numbers. They can also use graph paper and draw out the squares. After students have their lists, have them compare them with another classmate or two to see if they found the same numbers. Challenge students who have found patterns to make predictions for what number comes next, and then try to build or draw it to check that they're right.

Tips for the Classroom.

1. It's worth mentioning to your students that these numbers, which represent the number of little squares it takes to make a bigger square, are called square numbers.

2. While a good target for students is to build and find the first 10 square numbers, it's okay if not all students get all ten. It's also okay if students go higher — to 11 squared, 12 squared, etc. — without building every single square. In fact, it's great if they start finding patterns that help them predict what the larger square numbers will be.

Prompts and Questions

- How do you know that's a square?
- What makes a square a square?
- How do you know that this is the next largest square? (Did you use the last square as a starting point?)
- How did you count the tiles? (One by one, or some other way?)
- How do you know that you didn't make a mistake in your counting?
- Do you all agree that this is the correct count?

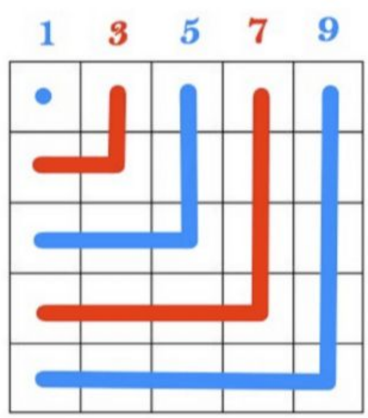
Closer

Bring the class together and have the students give you the numbers they found. The numbers should look like this:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Discuss patterns, and have students share patterns they found.

Some patterns students may have found include:
- Looking at the pattern of odds and evens (odd, even, odd, even, etc.)
- Noticing that the numbers are 1 × 1, 2 × 2, 3 × 3, 4 × 4, etc.
- Noticing how much each number in the pattern increases by. For square numbers, the pattern is +1, +3, +5, +7.

Adding L-shapes gives this pattern quite nicely. It's a nifty way to see how to build up to the next square.

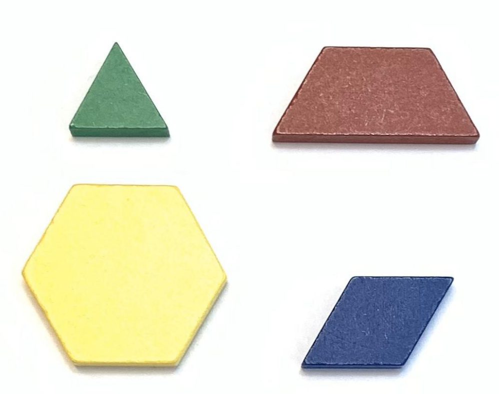


Leave students with the question: how can all these patterns be true at once?

Prompts and Questions

- What patterns did you notice?
- Is there a quick way to find how many squares you'll need if you know the side length of the square? For example, can you predict how many tiles you'll need to build an 11 by 11 square?

Pattern Block Scaling 1

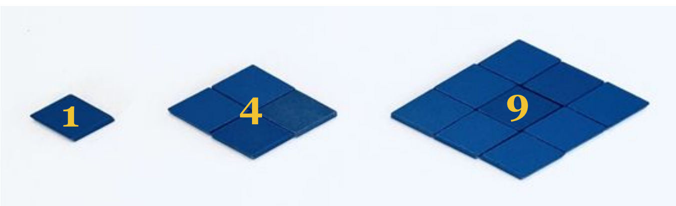


Materials and Prep
21st Century Pattern Blocks, paper, and pencil.

Motivating Question:
What patterns can we discover if we build bigger triangles from smaller triangles, or bigger trapezoids from smaller trapezoids, or any bigger versions of smaller shapes?

Launch
Remind the students that yesterday the class built bigger squares out of smaller squares, and recorded the list of squares numbers: 1, 4, 9, 16, 25, etc.

Tell students that today, they'll do the same thing with different shapes. In particular, they will use equilateral triangles, rhombuses, trapezoids and hexagons to explore the patterns that emerge from building bigger versions of a smaller shape. Make sure students are recording their work, since part of the goal is to see what new number patterns they get for each shape. As an example, ask students to try building larger rhombuses from smaller ones. What numbers do they get?



Launch Key Points

- Make sure students know how to check if they've built a bigger version of their shape by looking at side lengths.
- Students will need to record the number of shapes it takes to create their larger versions.

Work
Give students 30-40 minutes to build shapes of different sizes and have them write down their list of numbers. After students have their lists, have them compare them with another classmate or two to see if they found the same numbers. Challenge the students who have found patterns to make predictions for what number comes next, and then try to build or draw it to check that they're right.

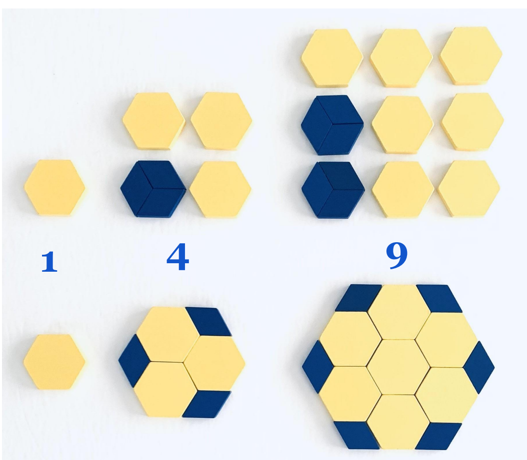
Anticipate that the students will have some misconceptions about what it means to be the same shape. *In this context, two figures have the same shape if the relationship between their sides is the same*. For example, all the sides of the rhombus block are the same length. If a larger shape doesn't have all four sides of the same length, it's not the same shape.

A rhombus, by definition, is a four-sided shape with all four sides the same length. For example, in the picture above, the shape with four rhombus blocks has the same shape as the individual rhombus, and the shape with nine does, as well.

The trapezoid is a little trickier. Three of its sides are the same length, and one is twice as long. To build a bigger version of this, make sure that the bigger version has three sides the same length, and the last side is twice as long.

There's a mathematical term for having the same shape (and not necessarily the same size): **similarity**.

It turns out that it's impossible to build bigger hexagons from smaller hexagons. Once students realize this, though, you can suggest using the smaller shapes to help build the bigger hexagons. This allows us to actually measure how many hexagons are in larger hexagons, if they use smaller blocks and think about them as fractions of hexagons.



Tips for the Classroom

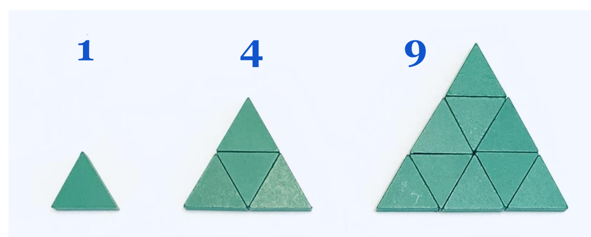
1. As needed, support students with making sure their bigger shapes have the same relationship between their sides.

2. While a good target for students is to build and find the first 10 figures, it's okay if not all students get all ten. It's also okay if students build beyond the first 10 figures without building every single shape. In fact, it's great if they start finding patterns that help them predict what the larger square numbers will be.

3. It's okay to see the patterns differently. For example, some students may count how many shapes there are in each hexagon (1, 6, 13, 24, 37, etc.) or count each type of rhombuses (0, 3, 6, 12, 18, etc.) and hexagons (1, 3, 7, 12, 19, etc.) separately.

Prompts and Questions

- How do you know that's a bigger version of a rhombus, a trapezoid, a triangle, a hexagon?
- What makes it a triangle, a rhombus, a trapezoid, a hexagon?
- How do you know that this is the next largest shape?
- Do you notice a pattern between shapes?
- Is your shape similar or symmetrical?
- Can you build a bigger version of a hexagon? Can you use smaller shapes to help you build a bigger hexagon?



Closer

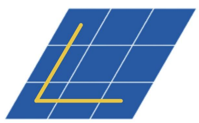
Bring the class together and have students share the numbers they found.

The astonishing result is that the same series of numbers appear no matter what shape was used. It's the square numbers again: 1, 4, 9, 16, 25!

Discuss how many of each shape it took to build the various larger versions and what conjectures and questions students have about their findings.

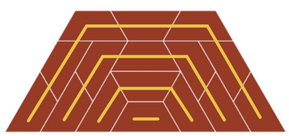
Some patterns students may have found include:

- Noticing that the number pattern alternates between odds and evens.
- Noticing that the numbers are 1 × 1, 2 × 2, 3 × 3, 4 × 4, etc. for the rhombuses.

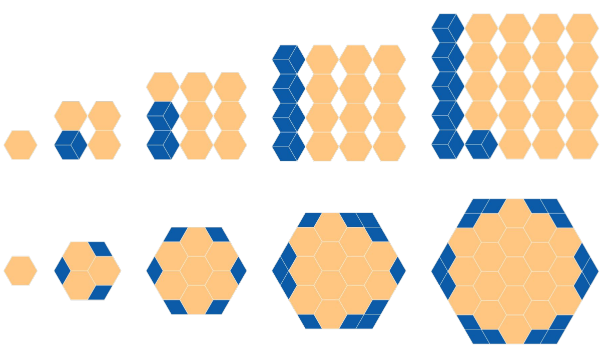


- Noticing how much each number in the pattern increases by. For square numbers, the pattern is +1, +3, +5, +7, ...

Adding the number of triangles in each row, or adding the number of surrounding trapezoids show this pattern quite nicely.



Leave the students with the question: how do the square numbers appear in the hexagon pattern?



Prompts and Questions

- Is the pattern discovered true for all shapes or just the once we've tried?
- Is it true only for pattern blocks?
- What if we had random triangles? Or a circle?
- Does it work for three dimensional shapes like cubes?

Pattern Block Scaling 2

Materials and Prep

21st Century Pattern Blocks, paper and pencil.

Motivating Question:

What patterns do you notice if you build bigger versions of smaller shapes that we haven't used yet?

Launch

Remind students of their findings from the previous lesson. They found that building bigger versions of the equilateral triangles, rhombuses, trapezoids, and hexagons formed square numbers: 1, 4, 9, 16, 25 ....

Tell students that today, they'll do the same thing with different shapes. In particular, they will use concave hexagons, right triangles, darts, and kites to explore the patterns that emerge from building bigger versions of a smaller shape. Make sure students are recording their work, since part of the goal is to see what new number patterns they get for each shape. As an example, ask students to try building larger concave hexagons. What numbers do they get?

After students find the numbers, pose a conjecture.

Conjecture.
All pattern blocks will produce square numbers only when building bigger versions of themselves.

Can students find a counterexample for this conjecture?

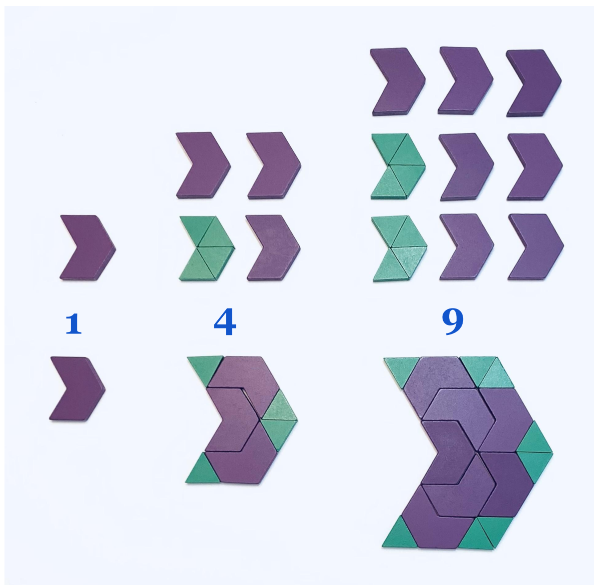
Launch Key Points

- Make sure students know how to check if they've built a bigger version of their shape by looking at side lengths.
- Students will need to record the number of shapes it takes to create their larger versions.

Work

Give student pairs 20-40 minutes to build shapes of different sizes and have them write down their list of numbers. After students have their lists, have them compare them with another pair of students to see if they found the same numbers. Challenge the students who have found patterns to make predictions for what number comes next, and then try to build or draw it to check that they're right.

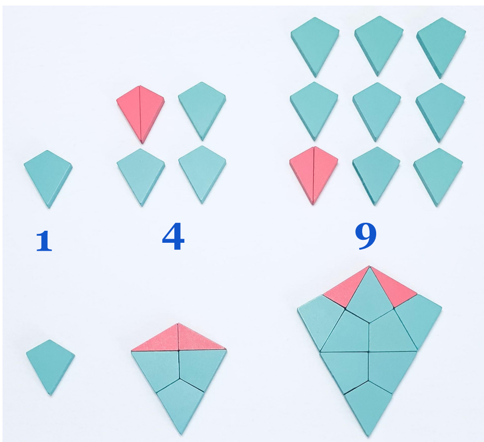
Anticipate that the work today will be more challenging as only the pink right triangle can be used to make a bigger version of itself using only pink triangles. Darts, kites and concave hexagons need other blocks added to make bigger versions of themselves.

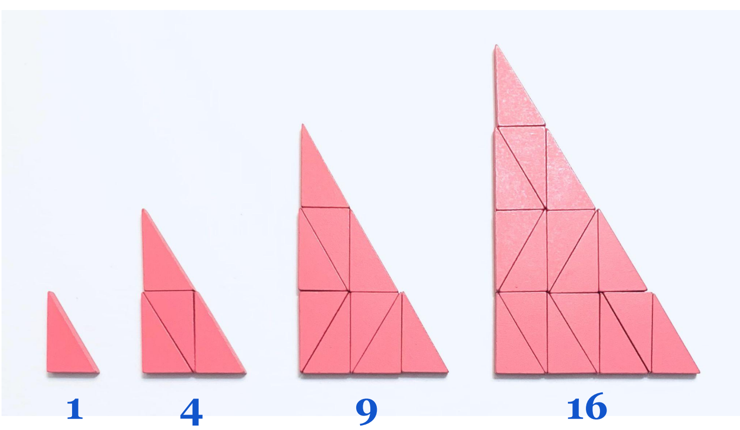


Work (continued)

Circulate the classroom and, help students extend their thinking, or get unstuck. Anticipate that the students will have some misconceptions about what it means to be the same shape. *In this context, two figures have the same shape if the relationship between their sides is the same.* For example, all sides of the concave hexagon are the same length. If a larger shape doesn't have all six sides of the same length, it's not the same shape.

The darts, kites, and right triangles are a little trickier.





Prompts and Questions

- How do you know that's a bigger version of a dart, a kite, a concave hexagon, a right triangle?
- What makes it a dart, a kite, a concave hexagon, a right triangle?
- How do you know that this is the next largest shape? Is there one you missed?
- Do you notice a pattern?
- Is your shape similar or symmetrical?
- Can you build a bigger version of a concave hexagon? Can you use smaller shapes to help you build a bigger concave hexagon?
- What if you placed the block here? What it you turned it around?

Tips for the Classroom

1. Students may make discoveries you're not expecting. Approach them with optimistic skepticism. If students discover something new and surprising, they should be able to convince you and their peers that their example holds up under scrutiny.

2. As needed, support students with making sure their bigger shapes have the same relationship between their sides.

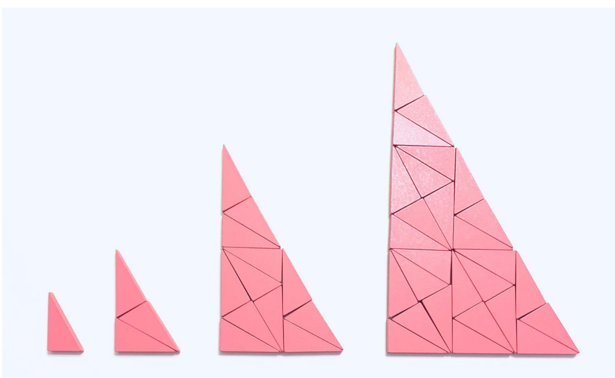
Closer

Bring the class together and have students share the numbers they found.

Discuss how many of each shape it took to build the various larger versions.

Revisit the conjecture.
All pattern blocks will produce square numbers only when building bigger versions of themselves.

Counterexample.
The right triangle can make a bigger version of itself with only three blocks, producing this pattern: 1, 3, 12, 27, 48. These are not square numbers.



1 3 12 27

If students found this counterexample, that's fantastic. If not, hopefully it's a huge surprise, and helps them realize that not all patterns can be trusted!

Prompts and Questions

- Is the pattern discovered true for all shapes that we've tried today?
- Is it true only for pattern blocks?
- Does it work for three dimensional shapes like cubes?